

# Transverse Flow of light clusters in Nuclear Reactions

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Cyclotron Institute REU 2010

## Introduction

The recent advances made in radioactive nuclear beam facilities allow us to study collisions between nuclei with unequal number of protons and neutrons. Studying these collisions would help us obtain information about the structure of nuclei with large isospin asymmetry and yield a better understanding of many astronomical processes such as nucleosynthesis in pre-supernova evolution of massive stars and the cooling process of proto-neutron stars (B. A. Li et al., Phys. Rep. 464,113 (2008)).

The final objective in the study of heavy-ion reactions induced by neutron-rich nuclei is to obtain information about the equation of state of asymmetric nuclear matter, specially its isospin-dependent term, i.e., the density dependence of the nuclear symmetry energy. Although nuclear symmetry energy at normal nuclear matter density is known to be about 30 MeV from the empirical liquid-drop mass formula, we lack the knowledge of its value at lower and higher densities.

Probes for density dependence of symmetry energy:

- 1) Collective flow
- 2) Isospin diffusion
- 3) Neutron skin thickness
- 4)  $\pi/\pi^+$ ,  $K^0/K^+$ , ...

## Nuclear Symmetry Energy

Various studies show that the energy per nucleon  $E(\rho, \delta)$  in asymmetric nuclear matter can be approximated by the parabolic function

$$E(\rho, \delta) \approx E(\rho, 0) + E_{sym}(\rho)\delta^2$$

where  $E(\rho, 0)$  is the equation of state of symmetric nuclear matter at density  $\rho$ ,  $E_{sym}(\rho)$  is the symmetry energy, and  $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$  is the isospin asymmetry.

Symmetry energy is defined as:

$$E_{sym}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \Big|_{\delta=0} \approx E(\rho, 1) - E(\rho, 0)$$

which is approximately the energy cost to convert all the protons in a symmetric nuclear matter into neutrons at the fixed density  $\rho$ .

## IBUU transport model

The isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) model has been shown to be useful in studying the dynamics and isospin effects in heavy-ion reactions. The BUU equation (G. F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988))

$$\begin{aligned} \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f \\ = - \frac{1}{(2\pi)^6} \int d^3 p_2 d^3 p_2' d\Omega \frac{d\sigma}{d\Omega} v_{12} \\ \times \{ [f_2 f_2' (1 - f_1)(1 - f_1') - f_1 f_1' (1 - f_2)(1 - f_2')] \\ \times (2\pi)^3 \delta^{(3)}(p + p_2 - p_1' - p_2') \} \end{aligned}$$

is solved by means of test particle method. It is based on the cascade model considering mean-field effects and quantum effects such as Pauli blocking and Fermi motion.

## Momentum-Dependent Interaction (MDI)

One of the most important inputs in our transport model is the single-nucleon potential. We used an isospin- and momentum-dependent single-nucleon potential derived from a Hartree-Fock approximation based on a Gogny-like effective interaction (C. B. Das et al., Phys. Rev. C 67, 034611 (2003))

$$\begin{aligned} U(\rho, \delta, p, \tau, x) = & A_u(x) \frac{\rho \tau'}{\rho_0} + A_l(x) \frac{\rho \tau}{\rho_0} \\ & + B \left( \frac{\rho}{\rho_0} \right)^\sigma (1 - x \delta^2) - 8\tau x \frac{B}{\sigma+1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \delta \rho \tau' \\ & + \frac{2C_{\tau\tau}}{\rho_0} \int d^3 p' \frac{f_\tau(r, p')}{1 + (p-p')^2/\Lambda^2} \\ & + \frac{2C_{\tau\tau'}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(r, p')}{1 + (p-p')^2/\Lambda^2} \end{aligned}$$

where  $\tau = 1/2$  (-1/2) denotes neutrons (protons);  $\tau \neq \tau'$ ,  $f_i(r, p)$  represents the phase-space distribution functions,  $A_u(x)$ ,  $A_l(x)$ ,  $B$ ,  $C_{\tau, \tau'}$ ,  $C_{\tau, \tau'}$ ,  $\sigma$  and  $\Lambda$  are constants;  $E(\rho_0)/A = -16$  MeV,  $E_{sym}(\rho_0) = 30.5$  MeV,  $K_0 = 212$  MeV, where  $\rho_0 = 0.16$  fm<sup>-3</sup>. The  $x$  is adjusted to vary the symmetry energy behavior at  $\rho \neq \rho_0$ .

The neutron (proton) single-nucleon potential can also be expressed as

$$U_n(p) \approx U_0 \pm U_{sym} \delta$$

where "+" and "-" are for neutrons and protons, respectively, and  $U_{sym}$  is the symmetry potential.

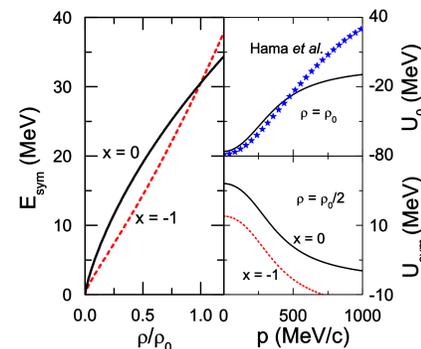


Fig. 1. (Left) Symmetry energy as a function of density for the MDI interaction with  $x = 0$  and  $x = -1$ . Isoscalar potential  $U_0$  (Top right) and symmetry potential  $U_{sym}$  (bottom right) as functions of momentum with  $x = 0$  and  $x = -1$ .  $U_0$  is consistent with the Schrödinger equivalent optical potential extracted by Hama et al. up to 500 MeV/c.

## Coalescence model

The coalescence model analyses the production of light clusters (deuteron, triton and <sup>3</sup>He) by evaluating the overlap of the wave functions of nucleons after heavy-ion collisions with the wave functions of light clusters (L. W. Chen et al., Nucl. Phys. A 729, 809 (2003))

$$\begin{aligned} \frac{dN_d}{d^3 p_d} = & G \int d^3 r_p d^3 p_p d^3 r_n d^3 p_n \delta^{(3)}(p_d - p_p - p_n) \\ & \times f_p(r_p, p_p) f_n(r_n, p_n) f_d^W(r_p - r_n, p_p - p_n) \end{aligned}$$

where  $G$  is the statistical factor with a value of 3/8 and

$$f_d^W = 8 \exp\left(-\left(\frac{r^2}{\sigma^2}\right) - (p^2 \sigma^2)\right)$$

is the Wigner function of deuteron with  $r = r_p - r_n$  and  $p = (p_p - p_n)/2$ , and the parameter  $\sigma$  is determined by the charge root-mean-square radius of the deuteron.

Similarly the Wigner phase-space density for triton or <sup>3</sup>He is given by:

$$f_t^W(^3\text{He}) = 8^2 \exp\left(-\frac{\kappa^2 + \lambda^2}{b^2}\right) \exp(-(p_\kappa^2 + p_\lambda^2)b^2)$$

where  $b$  is determined from the root-mean-square radii of triton or <sup>3</sup>He.  $\kappa$  and  $\lambda$  are the relative coordinates, and  $p_\kappa$  and  $p_\lambda$  are the relative momentum.

## Results

Input for the IBUU code:

- Target: <sup>70</sup>Zn
- Projectile: <sup>70</sup>Zn
- Impact parameter: 5.5 fm
- Test particles: 200
- Number of events: 100
- Energy: 35 MeV/A

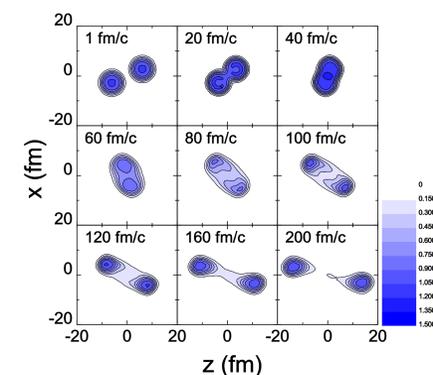


Fig. 2. Time evolution of the density profile in the reaction (x-z) plane.

We let the code run until 250 fm/c and analyze the final nucleon phase-space distribution. The free protons are determined when no other nucleons are within a space distance  $\Delta r = 3$  fm and momentum distance  $\Delta p = 0.3$  GeV/c. The deuteron, triton and <sup>3</sup>He are analyzed by the coalescence model.

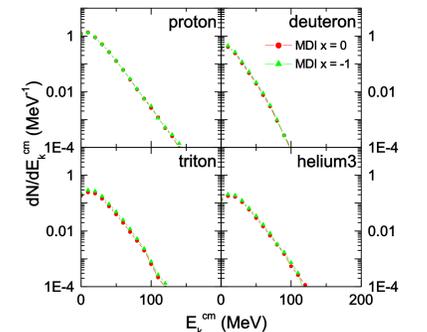


Fig. 3. Energy distribution of proton, deuteron, triton and <sup>3</sup>He from the MDI interaction with  $x = 0$  and  $x = -1$  in the center-of-mass frame.

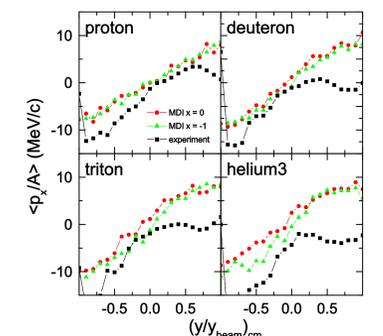


Fig. 4. Transverse flow as a function of rapidity of proton, deuteron, triton and <sup>3</sup>He from the MDI interaction with  $x = 0$  and  $x = -1$  compared with the experimental data (from Zachary Kohley dissertations).

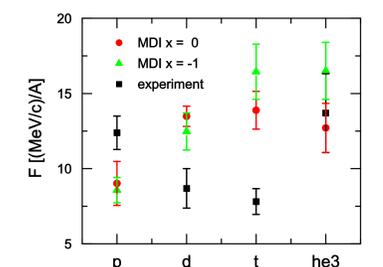


Fig. 5. Flow parameter ( $F = d\langle p_x/A \rangle/d[(y/y_{beam})_{cm}]$ ) of proton, deuteron, triton and <sup>3</sup>He extracted from the transverse flow within  $-0.3 < (y/y_{beam})_{cm} < 0.3$  from the MDI interaction with  $x = 0$  and  $x = -1$  compared with the experimental data.

## Conclusions

- The cluster multiplicity is slightly larger for stiff symmetry energy.
- The transverse flow of light clusters, from the IBUU model using MDI interaction along with the coalescence model, is smaller for proton and larger for deuteron, triton and <sup>3</sup>He compared to the experimental data.
- In order to get more obvious symmetry energy effects on the transverse flow, good statistics are required. Also, effects of the in-medium NN cross sections and other cluster formation mechanisms need to be studied.

## Acknowledgements

Cyclotron Institute  
Texas A&M University

Dr. Sherry Yennello

